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Linking Simple Economic Theory Models and the Cointegrated Vector AutoRegressive Model: Some Illustrative Examples

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Linking Simple Economic Theory Models and the Cointegrated Vector AutoRegressive Model - Some illustrative examples

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Abstract

This paper attempts to clarify the connection between simple economic theory models and the approach of the Cointegrated Vector-Auto-Regressive model (CVAR). By considering (stylized) examples of simple static equilibrium models, it is illustrated in detail, how the theoretical model and its structure and assumptions can be translated into a CVAR. We also see how the CVAR allows for explicit hypotheses about transitory dynamics, that could be relevant for assessing price rigidity, and hence, "the length of the short run" - a controversial issue in traditional macroeconomics. Moreover, it is demonstrated how other controversial hypotheses such as Rational Expectations can be formulated directly as restrictions on the CVAR-parameters. A simple example of a "Neoclassical synthetic" AS-AD model is also formulated. Finally, the partial- general equilibrium distinction is related to the CVAR as well. Further fundamental extensions and advances to more sophisticated theory models, such as those related to dynamics and expectations (in the structural relations) are left for future papers.

1 Introduction[1]

The Cointegrated Vector AutoRegressive (CVAR) model, has become a popular approach in applied economic analyses of time series. Often this model can seem hard to grasp for non-expert-economists. Many unfamiliar and technical concepts, such as common trends, cointegrating relations and attractor set, etc. are introduced to economists taking their first course in the CVAR model. Even after having completed such courses it still might seem hard to interpret results in applied papers, and some kind of "black box" status is likely to emerge, probably providing the soil for criticism. Hence, on the surface, the tools, terminology and concepts from the theory of the CVAR seem quite far from those of traditional economic theoretical analysis.

[1] I would like to thank the following for valuable and detailed comments (alphabetically by surname): Massimo Franchi, Søren Johansen, Katarina Juselius and Christin Tuxen.

By the use of some simple examples, this paper demonstrates that quite the opposite is the truth!

The main purpose of this paper is to clarify the link between economic theory models and the CVAR. The overall idea is therefore simply to take economic theory models, and suggest their counterpart in terms of a CVAR. In practice, this means focussing on a few "representative" models, that each roughly captures the basic common structure of a wealth of specific economic theory models. The present paper takes a few steps in this direction by considering some examples of simple static theory models. We do not to question derivations from microeconomic foundations, but rather focus on the rough and basic implications of static theory models. For our purpose, it does not matter whether the aggregate labour market is characterized by a Price-Setting- and a Wage-setting relation, describing imperfect competition, or by competitive Demand and Supply relations, both market structures imply two structural crossing curves that are shifted up and down by exogenous variables[2]. Clearly, it is understood that, the present paper does not attempt to develop new theory models neither, nor develop statistical tools, but rather tries to clarify the connection between existent theory models and the CVAR. It is intended to be pedagogical by using illustrative examples and keep it as simple and explicit as possible. The paper is essentially meant for non-expert economists, however with some knowledge (graduate level) of the CVAR, interested in applying the method.

Hopefully, this exercise will demonstrates some of the great potential that the CVAR has as a statistical framework for analyzing economic phenomena *on the basis of economic theory models,* in particular. As will become evident, the hypothetical connection between, say the simple economic demand and supply cross, and its counterpart in the CVAR, is extremely close and immediate. The exercise also demonstrates the potential of the CVAR as a promising framework for answering some of the (most) central questions in macroeconomics, such as "how long the short run is", when partial equilibrium is sufficient or when imposition of general equilibrium is needed, etc.?

Admittedly, the theory models considered here are *very* simple, and it should be underscored that they are not meant to characterize what modern applied economists actually are working with, but rather form a simple point of departure from which we can advance gradually. Nevertheless, such simple static models and various variants thereof still make up a great part of modern economic curriculum, as well as of the everyday toolbox of the "typical economist".

To be precise we start by considering an example of a simple model that could correspond to a *static partial* equilibrium model - say the usual Demand and Supply cross. This model is introduced in Section 3 and translated into a CVAR in Section 4. Before this, some simple tools and concepts from the analysis of the CVAR are briefly refreshed in Section 2. In Section 4 we also consider how the CVAR can model transitory dynamics of great economic interest (such as staggering price setting, say) - something the simple static theory model does not say anything about explicitly. In light of these simple examples we also formulate restrictions on the CVAR corresponding to the interesting case which could be referred to as "Model Consistent, or

[2]Obviously this distinction matters for efficiency considerations.

Rational Expectations in the presence of unanticipated shocks". Based on this it is illustrated how policy ineffectiveness implied by "Rational Expectations" can be tested in the CVAR. Moreover, we look at a simple example of extending the basic CVAR with one lag, which seems useful in relation to a simple static AS-AD model, where we state the restrictions needed for the model to have "Classical" long-run properties while "Keynesian" short-run properties. Finally, Section 4 considers an important extension in economics of the simple theory model, namely that of imposing general equilibrium, and looks at a simple "two-market" example. In Section 5 we briefly state some typical practical problems in the CVAR, and mention the most obvious extensions of the theory model, while Section 6 summarizes and concludes.

2 The Cointegrated VAR model - some basic tools

It is taken for granted that the reader has some knowledge of the usual I(1) cointegration analysis, and is otherwise referred to Johansen (1996) and Juselius (2006) for technical details and applications. As in the spirit of the present paper, we shall try to keep it as simple as possible, and focus only on the tools that are relevant for the time being.

We consider here the *reduced form* VAR model (i.e., no simultaneous effects), in ECM reparameterization

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \ \Delta x_{t-i} + \phi D_t + \varepsilon_t, \quad t = 1, ..., T. \tag{1}$$

where x_t is $p \times 1$, $\Pi \equiv \Sigma_{i=1}^{k} \Pi_i - I_p$ and $\Gamma_i \equiv -\Sigma_{j=i+1}^{k} \Pi_j$ are $p \times p$, and Φ is $p \times d$, so that the D_t is a $d \times 1$ term of d deterministic components. The integer k is the number of lags in the VAR model, $x_t = \Pi_1 x_{t-1} + ... + \Pi_k x_{t-k} + \varepsilon_t$. The innovations, ε_t are $N_p i.i.d.(0, \Omega)$, with Ω being diagonal, and the initial values, $x_{1-k}, ...x_0$, are as usual treated as fixed. For conditions below we define $\Gamma \equiv I - \Sigma_{i=1}^{k-1} \Gamma_i$. Finally, the characteristic equation corresponding to (1), can be written as

$$|A(z)| = \left| I - \sum_{i=1}^{k} \Pi_i z^i \right| = 0 \tag{2}$$

To avoid cases of less empirical relevance we shall assume the following about the roots of $A(z)$

$$\text{if } \ |A(z)| = 0, \ \text{then either } \ |z| > 1 \text{ or } z = 1 \tag{3}$$

Given (3), the I(1) model with cointegration is defined by the rank conditions

$$r(\Pi) = r < p \text{ and } r(\alpha'_{\perp} \Gamma \beta_{\perp}) = p - r \tag{4}$$

where $r(\cdot)$ denotes the rank operator (Johansen 1996). If the process has a unit root it implies that Π has reduced rank, r, and therefore can be "singular value decomposed" into full rank $p \times r$ matrices α and β, so $\Pi = \alpha\beta'$. The matrices $\alpha_{\perp}$ and $\beta_{\perp}$ are the $p \times (p - r)$ orthogonal complements, i.e. defined by $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$, implying that the second condition requires full rank on $\alpha'_\perp\Gamma\beta_\perp$, preventing higher orders of integration.

As the economic theory models we are considering do not say anything about transitory dynamics of adjustment, but are solely concerned with (short-run or long-run[3]) *equilibrium movements,* a natural point of departure for us would be a VAR with one lag - VAR(1). In a VAR(1) all transitory dynamics are adjustment to long-run (cointegrating) relations.

We also assume that the only deterministic term present is a constant, which is restricted to the span of α. Thus our initial simple statistical model is given by

$$\Delta x_t = \alpha\beta' x_{t-1} + \phi + \varepsilon_t, \quad \phi = \alpha s, \quad s \in R^r \tag{5}$$

The *Moving Average representation* of a VAR(1) model like (5) can be written explicitly in terms of the parameters $\alpha, \beta, \alpha_\perp$ and $\beta_\perp$

$$\begin{aligned} x_t &= C\sum_{i=1}^{t}\varepsilon_i + \sum_{i=0}^{\infty} C_i^*(\alpha s + \varepsilon_{t-i}) + Cx_0 \\ C &= \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp, \quad C_i^* = \alpha(\beta'\alpha)^{-1}(I_r + \beta'\alpha)^i\beta' \end{aligned} \tag{6}$$

where the stationary components in the process has been assigned appropriate initial values (See see proof of Theorem 2.2 in Johansen 1996). Note that, since the constant term is restricted to the span of α there is only a non-cumulated effect from the constant, that does not disappear in the cointegrating relations. That is, we have no (cancelling) deterministic trends in the levels and only non-zero levels of the cointegrating relations (See Juselius 2006, Chapter 6).

Given (6) the *impulse response function* has the representation

$$\frac{\partial E(x_{t+h} \mid x_t)}{\partial \varepsilon_t} = C + C_h^* \rightarrow C \text{ for } h \rightarrow \infty \tag{7}$$

That $C_h^* \rightarrow 0$ follows from the I(1) assumption, in that the eigenvalues of $(I_r + \beta'\alpha)$ are those from Π_1 that are less than 1 in modulus.

The pushing forces of the system are described by the *common (stochastic) trends* (CT), given by

$$\mathrm{CT} \equiv \alpha'_\perp\Sigma_{i=1}^t\varepsilon_i \tag{8}$$

The *attractor set* is, in this case given by

$$A \equiv \{x \in R^p \mid \beta' x = -s\} \tag{9}$$

Usually we split up the stochastic trend, $C\Sigma_{i=1}^t\varepsilon_i$, in the common trends, $\alpha'_\perp\Sigma_{i=1}^t\varepsilon_i$, and the so-called *loadings matrix,* in our case given by

$$L \equiv \beta_\perp(\alpha'_\perp\beta_\perp)^{-1} \tag{10}$$

[3] "Short-run"/ "long-run" in the macroeconomic theoretical sense.

which tells us exactly how each of the $p-r$ common trends affect the individual variables of the process.

We have now presented some of the basic tools and concepts from the I(1) cointegration analysis. In the next section we introduce our first simple example of a theory model. Subsequently this is then related to the results from this section (Section 4.2).

3 A simple static theory model

We consider first an example of a simple but typical economic model. On the one hand, it is simple since it is *static*, i.e. describes contemporaneous relations between variables, and only considers a few variables - two endogenous and two exogenous. On the other hand, it is typical, since a wealth of economic models has the same type of basic structure or algebraic representation. In this sense, the example is, in fact, relatively far-reaching, and could be consistent with the typical Demand and Supply analysis of some market, the static IS-LM and AS-AD models, or, say, the more modern Wage-Setting/Price-Setting model of imperfect competition in the aggregate labour market. In passing, note that, whereas the first type of these models, the Demand and Supply models, are thought of as *partial equilibrium* models, the rest are, in fact, *general equilibrium* models, where two relations, say, the AD and the AS relations, imply equilibrium in several markets (the aggregate goods-, labour- and financial market). Nevertheless, roughly speaking, all these models have the same geometric representation (see Figure 1).

We call the two endogenous variables x_1 and x_2, and the two exogenous variables x_3 and x_4. Suppose that the model relates these variables, according to the two following *structural relations* for the endogenous variables

$$\begin{aligned} x_{2t}^d &= a_0 - a_1 x_{1t} + a_2 x_{3t} \\ x_{2t}^s &= b_0 + b_1 x_{1t} + b_2 x_{4t} \end{aligned} \tag{11}$$

Where all parameters are positive, for notational convenience. These relations are typically *behavioral relations* describing *intentions* of the agents in the economy. The superscripts, d and s indicates that we are going to study demand and supply intentions, merely for convenience.

As is evident, the model is linear. This might not be very realistic, and in general we shall interpret the xs as logarithmic, so that we have log-linearity. This log-linearity might be interpreted as a local approximation. However, here we regard the theory model as being log-linear[4]. Thus, the original variables are strictly positive valued, and the logarithmic formulation in (11), ensures that predictions of the original variables based on the empirical model counterpart to (11) obey this[5].

The model is illustrated in Figure 1 in the positive quadrant (but in general $(x_1, x_2) \in R^2$).

[4]This is to abstract from the problems that may arise when making log-linear approximations of relations between nonstationary variables.

[5]That is, the empirical model should be dataadmissable (Hendry and Richard 1982)

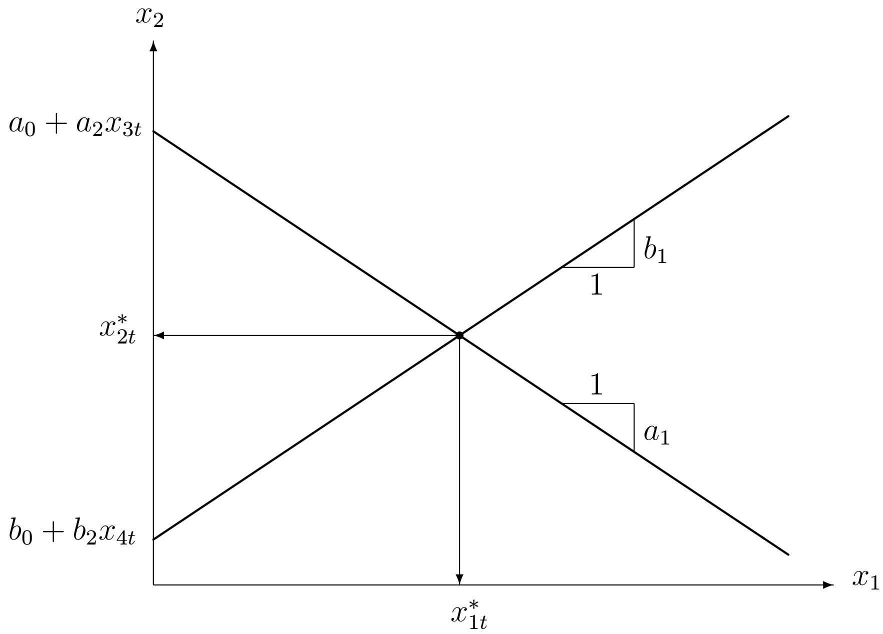

Figure 1: The theory model for given values of the exogenous variables, x_3 and x_4.

In general, given the log-linear formulation in (11) an *interior* economic equilibrium, $x_{2t}^d = x_{2t}^s$, *exists and is unique* provided that $b_1 \neq -a_1$, i.e. that the curves in Figure 1 are not parallel[6]. This assumption is of course fulfilled since b_1 and a_1 are both positive by assumption. If $b_1 = -a_1$, this would imply either no solution or infinitely many (the lines coincide).

The economic equilibrium is,

$$\begin{aligned} x_{1t}^* &= \frac{(a_0 + a_2 x_{3t}) - (b_0 + b_2 x_{4t})}{a_1 + b_1} \\ x_{2t}^* &= \frac{b_1(a_0 + a_2 x_{3t}) + a_1(b_0 + b_2 x_{4t})}{a_1 + b_1} \end{aligned} \tag{12}$$

Note that, generally in this paper, by "equilibrium" is meant that all intentions are realized, not necessarily market clearing.

Based on the structural relations and the equilibrium assumption, economists typically start analyzing the *equilibrium effects* on the endogenous variables of changes in the exogenous ones. These effects can be obtained as the partial derivatives of the equilibrium values in (12) with respect to the exogenous variables, $\frac{\partial x_{kt}^*}{\partial x_{lt}}$, $k = 1, 2$, $l = 3, 4$, and are referred to as the *comparative static effects*.

To enhance intuition, we can think of a demand and supply analysis of the chicken market, throughout, where the first relation in (11) is the (inverse) demand curve, and the second, the (inverse) supply curve. The two endogenous, x_{1t} and x_{2t} are quantity and price of chicken respectively, and x_{3t} is the price of other meat, while x_{4t} is some input price, so that a rise in x_{4t} shifts the supply curve upwards by raising production costs. The assumption, $a_2 > 0$, corresponds to the claim that chicken and other meat are, to some degree, substitutes (substi-

[6] Or more generally, that the determinant of the coefficient matrix to the system in (11) is not equal to zero.

tution effect dominates). Of course, realistically, there are many other variables affecting the demand and supply curves, such as income and other prices etc., but for the present purpose their inclusion would not enhance insight further, so we abstract from these, and imagine these variables have been relatively constant over the period of analysis.

Thus, we are considering a *static partial equilibrium* model of the chicken market. The idea is now to take this model, illustrated in Figure 1, and its basic assumptions and terminology, such as the equilibrium assumption, the structural relations, the exogenous/endogenous division, the comparative statics etc., and relate them to the Cointegrated VAR model, and its concepts from Section 2 - the cointegration relations, common trends, attractor set, impulse response function, and long-run effects etc. It will become evident that there is a completely natural correspondence.

4 Collecting the pieces - relating the economic model and the CVAR

The typical comparative static analysis of the model in Section 3 corresponds to implicitly conducting an experiment of thought like the following: From an initial equilibrium position, we imagine an *isolated* permanent shock to one of the exogenous variables, and then ask what has happened to the endogenous variables *after* all adjustment has taken place, i.e. when we are in the new equilibrium. We thus make two important simplifying assumptions. The first is that, the shock occurs in the absence of any further shocks whatsoever (to the other variables as well as to the variable in question) and the second is that we completely abstract from what happens on the way to the new equilibrium (during transition). There is no explicit time axis, and we are only considering what the effect will be *eventually.* As mentioned these abstractions mathematically correspond to taking partial derivatives of the equilibrium position with respect to the exogenous variables.

While such partial effects are what economists are interested in primarily, say for economic policy guidance, this is, of course, not what is usually observed in reality. All sorts of anticipated as well as unanticipated shocks happen each period affecting the variables of interest through a complicated dynamic structure, implying feedback effects, adjustment might be very persistent etc., and a serious empirical model cannot abstract form this. So the question is, how can we relate the simple *static* theory model to a dynamic empirical model like the CVAR?[7]

First of all, we can interpret the two structural relations in (11) as *contingent* plans (presumably derived from optimization). This means that agents have, or rather, act *as if* they had, made a plan once and for all (ex ante), to be implemented after having observed the outcome of some conditional variables (ex post) (Hendry 1995). For example an individual demand relation could capture a plan like, "*if* price turns out to be X_t, *then* I shall demand Y_t".

[7] According to Hendry and Juselius (2000), Sargan was relating static theory relations to dynamic empirical models (See Sargan 1964).

If a contingent plan dictates that a simultaneous or static relation is optimal, say, current consumption depends on current income, then it is natural that *anticipated* "shocks" or changes in the exogenous variables (income) at time t lead to changes in the planned variables at time t (consumption demand) - not before not after. On the other hand, if the shocks are *unanticipated,* that same contingent static plan would naturally imply adjustment in the *next* period (or later)[8].

Whether the actual values in the following period will coincide with the planned values depends on many things: In general, they will *not,* since perfect foresight naturally seems unrealistic, and one can argue that the difference is therefore, at the minimum, an (unanticipated) zero mean independent term (See below). However, apart from this new unanticipated shock, in the short run, the plans might not be equivalent to the structural relations in (11), but rather involve gradual adjustment towards these, say in the presence of adjustment costs (menu costs say). In such cases, one might modify, and call the structural relations *long-run* contingent plans. Even if the plans do not imply that gradual adjustment is optimal, other factors, such as physical and informational restrictions, sequential and uncoordinated interactions between buyers and sellers are likely to prevent the optimal plans of *all* agents (i.e. equilibrium) to be realized.

Note that, just by combining these simple *static* contingent plans, *and* unforeseen or unanticipated shocks, we have essentially formulated an Error-Correction-Mechanism (ECM): Agents react to the deviation from equilibrium in the previous period (Davidson, Hendry, Srba, and Yeo 1978). This is shown explicitly in Subsection 4.1.

Let us now relate this to the economic cross in Figure 1, with the contingent demand and supply curves. Each time period we imagine that *unanticipated* shocks to the exogenous variables occur, and change the positions of the curves, and hence the corresponding equilibrium position (x^*_{1t}, x^*_{2t}). In the subsequent period, $t+1$, suppliers and demanders react to these shocks, and start adjusting in accordance with their contingent plans. In this way, the equilibrium position in the previous period, (x^*_{1t}, x^*_{2t}), acts as a pulling force upon the observed point, (x_{1t+1}, x_{2t+1}), in the sense, that if there were no further shocks from period $t+1$, and onwards, the observed point in all future periods would gravitate towards the now constant equilibrium position (x^*_{1t}, x^*_{2t}). This hypothetical absence of any other shocks resembles the comparative static way of thinking.

One should note that unanticipated shocks to the endogenous variables also happen due to stationary unsystematic unmodelled factors. However, the important difference between the unanticipated shocks to the exogenous variables, and the unanticipated shocks to the endogenous ones, is that the former have permanent (long-run) effect on the endogenous variables, as they change the position of the curves, whereas the latter have only transitory effects as the curves are unaffected.

It seems natural to assume that all these unanticipated shocks are uncorrelated and zero mean, since if they were not, an anticipated component of the shocks would remain, which

[8]It is assumed that the periods are sufficiently short.

the agents probably would have incorporated into their plans in the first place. If we further assume that the shocks are *normal and uncorrelated* over time, the above set up (the *linear static* contingent relations in Figure 1 combined with such *unanticipated* shocks) will coincide formally with the CVAR (See Hendry 1995, Chapter 6). This is because in the CVAR we have that

$$x_t = E_{t-1}[x_t \mid x_{t-1}, x_{t-2}, ..., x_{t-k}] + \varepsilon_t \tag{13}$$

where $\varepsilon_t \equiv x_t - E_{t-1}[x_t \mid x_{t-1}, x_{t-2}, ..., x_{t-k}]$ and $E[\cdot]$ is the mathematical expectation. Under multivariate normality of x, $E[\cdot \mid \cdot]$ is *linear* in the past, $x_{t-1}, x_{t-2}, ..., x_{t-k}$, and ε_t is a normal uncorrelated term. The ECM-form of the CVAR (i.e. 1) is just the reparameterization of this that parallels the above theoretical set up.

We now try to illustrate this.

4.1 A preliminary example

Before we consider the model from Section 3 (Figure 1), let us concrete the above by considering an even simpler case of this theory model in order to show explicitly how the contingent plans from the theory model combined with unanticipated shocks coincide with the ECM reparameterization of the CVAR.

Consider the vector $x_t' = (c_t, p_t, y_t)$, where c is real consumption, p the real price of consumption and y real income (all in logs). Our theory model is that the consumption demand, as a function of price and income, is given by the static relation

$$c^p(y, p) = d_0 + d_1 y - d_2 p \tag{14}$$

and that y and p are both exogenous. As before, all parameters are positive, and the superscript (p) refers to a (contingent) plan. That p is exogenous is interpreted as a horizontal inverse supply curve or a vertical supply curve as depicted in Figure 2.

Now, imagine that the economy has been in the same equilibrium, E_0, up to and including period $t-1$. So, all plans have been realized, that is $c_\tau = c^p(y_\tau, p_\tau) \equiv c_\tau^p = d_0 + d_1 y_0 - d_2 p_0 = c_0$, for $\tau = 0, ...t-1$. Then, in period t, an *unanticipated* "supply shock", i.e. a price shock, occurs. The shock is assumed to be permanent. To begin with, let us assume that this is the only thing that happens. The shock is illustrated in Figure 2. The thing to note is that due to the fact that the shock is unanticipated, consumption does not react in period t. Since p is exogenous this implies that the actual position of the economy (the price-consumption point) jumps from E_0 to A in period t. In period $t+1$ however, the consumer has realized the price change. Since, at the given price, p_t, the (optimal) consumption according to the contingent plan is $c_t^p = c^p(y_t, p_t) = d_0 + d_1 y_t - d_2 p_t = d_0 + d_1 y_0 - d_2 p_t < c_t$, the consumer will adjust consumption in accordance with this. If she is not prevented from doing this in any way, if there are no adjustment costs, and no other consumption determinants change at all (remember we ignore

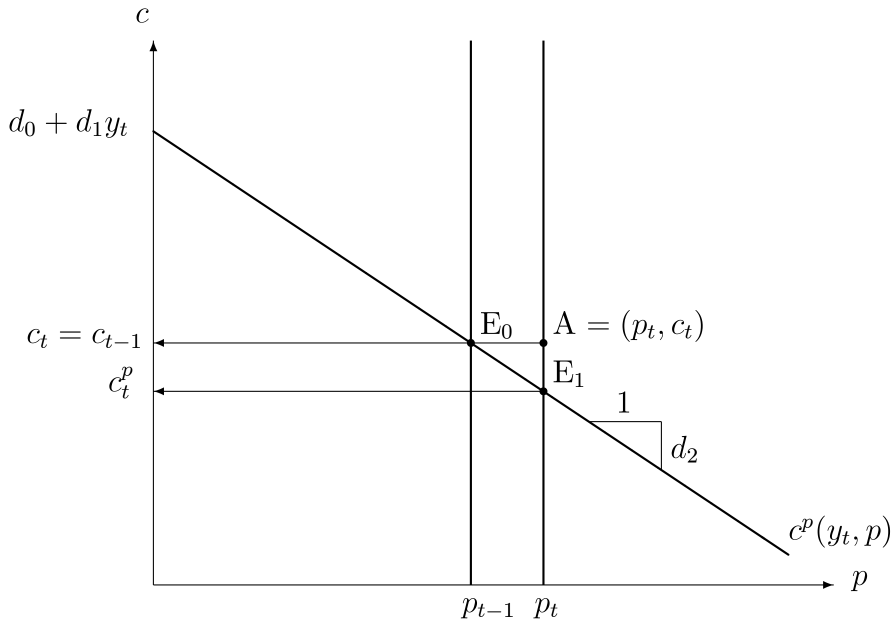

Figure 2: Consumption demand and supply.

all other shocks), the actual consumption will be the optimal one. That is,

$$c_{t+1} = c_t^p \tag{15}$$

and in period $t+1$ we jump from A to E_1, and consumption has been brought into balance again (See Figure 2).

Even if y and p determine the bulk of consumption it seems unlikely that consumption is going to be exactly what is planned. There might be many other (less important) factors influencing consumption which change unexpectedly. Therefore, it seems reasonable to add an error term in (15),

$$c_{t+1} = c_t^p + \varepsilon_{1t+1} \tag{16}$$

As mentioned, it seems natural that the error term is unsystematic, say is zero-mean and uncorrelated over time, as if this were not the case, some anticipated part would remain, the agent would make systematic mistakes, which seems hard to justify when actual consumption is chosen by the consumer herself (Hendry 1995, Chapter 6). For example, if the mean of the error were a positive constant this would be reflected in a higher intercept of the consumption function, so that the original contingent plan would not describe behavior adequately in the first place. Thus, we have that $E_t[c_{t+1} \mid c_t^p] = c_t^p$. Note that, this implies that $E_t[\varepsilon_{1t+1} \mid c_t^p] = 0$, which of course can be questioned. Finally, on the basis of the Central Limit Theorem we can argue that the error should be normal.

Now, in (16) subtract c_t on both sides to get

$$\Delta c_{t+1} = -(c_t - c_t^p) + \varepsilon_{1t+1} \tag{17}$$

This is essentially the error-correction-mechanism, with an adjustment coefficient α_1 equal to

-1.

If we further insert the expression for the contingent plan into (17), lag the expression once just for convenience, and assume that permanent shocks, ε_{3t}, to y_t also happen (demand shocks)[9], and that, like the consumption shocks, ε_{1t}, shock to prices, ε_{2t}, and ε_{3t} are normal, zero-mean and uncorrelated, we get

$$\begin{aligned} \Delta c_t &= -(c_{t-1} - (d_0 + d_1 y_{t-1} - d_2 p_{t-1})) + \varepsilon_{1t} \\ \Delta p_t &= \varepsilon_{2t} \\ \Delta y_t &= \varepsilon_{3t} \end{aligned} \tag{18}$$

where $\varepsilon_t \sim N.i.i.d(0, \Omega)$. As mentioned the theory model will usually imply that Ω is diagonal, in this context.

From (18) it is clear that this coincides with the ECM-reparameterization of the CVAR with one lag, cointegration rank equal to one etc. (See the next Subsection), derived from an *multivariate normal* I(1) process, x_t, fulfilling some basic statistical assumptions, concerning memory of the process etc. Note, that the contingent plan(s) must be (approximately) linear in order for the theoretical model plus unanticipated shocks to coincide with the CVAR.

The relevant matrices, α, β and their orthogonal complements, the C - matrix etc., of this model are easily derived but not stated here, as we consider a model in Section 4.4 that has the same structure. Instead, let us now consider the slightly more general model from Section 3 (Figure 1) and derive and interpret all the relevant matrices.

4.2 The static partial equilibrium model from Section 3

Obviously the theory model from Figure 1 implies that we consider a four-dimensional VAR. As argued previously, a natural starting point is one lag, and for convenience we have allowed for a restricted constant. The two structural relations in equation (11) correspond to the two normalized cointegration vectors $\beta_1 = (a_1, 1, -a_2, 0)'$, and $\beta_2 = (-b_1, 1, 0, -b_2)'$, where, as before, all parameters are positive, which is assumed in the rest of this section.

The theoretical exogeneity of x_{3t} and x_{4t} will in general correspond to these variables being strongly exogenous in the CVAR. In a VAR model with one lag only, strong and weak exogeneity coincide.

We shall also assume that the all non-zero $\alpha_{ij}s$ are reflecting *equilibrating* economic behavior, and together fulfill the restriction that $|\text{eig}(I_r + \beta'\alpha)| < 1$ given β.

[9] In general, in each period shocks to all three variables occur (unrelated), instead of just the isolated shock to p as in Figure 2. The equilibrium error is in general $\varepsilon_{1t} + d_2\varepsilon_{2t} - d_1\varepsilon_{3t}$, in this model, corresponding to an extra shift in the demand curve in Figure 2, $d_1\varepsilon_{3t}$, as well the consumption shock, ε_{1t} in addition to the price shock illustrated.

Given these assumptions our model is (5) with

$$\alpha = \begin{pmatrix} -\alpha_1 & \alpha_3 \\ -\alpha_2 & -\alpha_4 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \ \beta = \begin{pmatrix} a_1 & -b_1 \\ 1 & 1 \\ -a_2 & 0 \\ 0 & -b_2 \end{pmatrix} \text{ and } \phi = \alpha s, \ s = \begin{pmatrix} -a_0 \\ -b_0 \end{pmatrix} \tag{19}$$

and $x_t' = (x_{1t}, x_{2t}, x_{3t}, x_{4t})$.

For illustration we can write this equation-wise as,

$$\begin{aligned} \Delta x_{1t} &= -\alpha_1(x_2 - (a_0 - a_1 x_1 + a_2 x_3))_{t-1} + \alpha_3(x_2 - (b_0 + b_1 x_1 + b_2 x_4))_{t-1} + \varepsilon_{1t} \\ \Delta x_{2t} &= -\alpha_2(x_2 - (a_0 - a_1 x_1 + a_2 x_3))_{t-1} - \alpha_4(x_2 - (b_0 + b_1 x_1 + b_2 x_4))_{t-1} + \varepsilon_{2t} \\ \Delta x_{3t} &= \varepsilon_{3t} \\ \Delta x_{4t} &= \varepsilon_{4t} \end{aligned} \tag{20}$$

We note, that, in this case, it is always possible to write the model so that each endogenous variable adjusts to one relation only, but from (20) we see that such relation is a combination of the theoretical relations, unless for example $\alpha_3 = \alpha_2 = 0$, say. So, for interpretational reasons we use the representation in (20). This representation shows how the endogenous variables adjust when they are outside equilibrium, and that each endogenous variable adjusts to deviations from both theoretical relations, in general.

Given the matrices in (19) the orthogonal complements are given by

$$\alpha_\perp = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \beta_\perp = \begin{pmatrix} \frac{a_2}{b_1+a_1} & -\frac{b_2}{b_1+a_1} \\ \frac{a_2}{1+\frac{a_1}{b_1}} & \frac{b_2}{1+\frac{b_1}{a_1}} \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{21}$$

which result in the following common trends and loadings matrix

$$\text{CT} = \begin{pmatrix} \Sigma_{i=1}^t \varepsilon_{3i} \\ \Sigma_{i=1}^t \varepsilon_{4i} \end{pmatrix}, \ L = \begin{pmatrix} \frac{a_2}{b_1+a_1} & -\frac{b_2}{b_1+a_1} \\ \frac{a_2}{1+\frac{a_1}{b_1}} & \frac{b_2}{1+\frac{b_1}{a_1}} \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{22}$$

using equations (8) and (10).

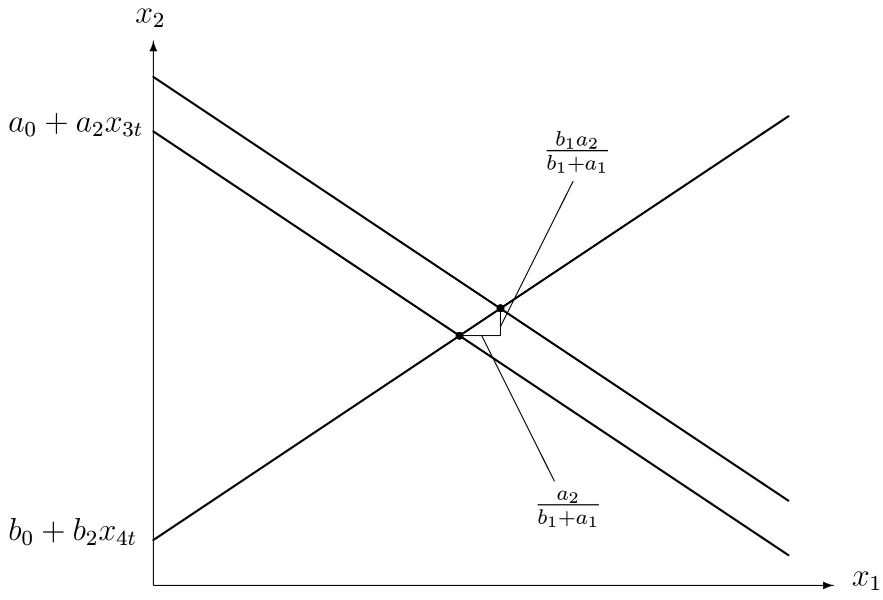

Figure 3: Illustration of the long run impact of a unit rise in ε_3 (positive demand shock) on the endogenous variables, x_1 and x_2.

From this, the long-run C matrix from (6) is given by

$$C = \begin{pmatrix} 0 & 0 & \frac{a_2}{a_1+b_1} & -\frac{b_2}{a_1+b_1} \\ 0 & 0 & \frac{a_2}{\frac{a_1}{b_1}+1} & \frac{b_2}{\frac{1}{a_1}b_1+1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

First note that, since we have assumed that (3) is fulfilled, the I(1) condition is fulfilled since $\alpha'_\perp \beta_\perp = I_2$, the 2×2 identity matrix which of course has rank $p - r = 2$. As we can see from (22) the common trends correspond to the cumulated shocks to the (theoretically) exogenous variables, x_{3t} and x_{4t}. These determine the long-run movement of the endogenous variables. In terms of the economic cross in Figure 1, each shock to x_{3t} and x_{4t}, pushes one of the curves up or down, implying a permanent effect so that the sum of all these shocks roughly determines the positions of the curves at time t.

As mentioned, the loadings matrix, L, in (22) shows how the common trends affect the variables, in particular, the endogenous variables. The interpretation of the elements in L is completely intuitive if we combine it with the economic cross in Figure 1. Consider a unit rise in ε_{3t} (which of course corresponds to a unit rise in $\Sigma_{i=1}^{t}\varepsilon_{3i}$). According to L in (22) this will have a long-run impact on x_1 of $\frac{a_2}{b_1+a_1}$ units and on x_2 of $\frac{a_2}{1+\frac{a_1}{b_1}}$ units. In terms of the economic cross this corresponds to an unanticipated unit shock to the price of other meat, x_{3t}, which shifts the demand curve upwards by a_2 units, eventually resulting in a rise in the equilibrium value of x_1 and x_2 of exactly these magnitudes, $\frac{a_2}{b_1+a_1}$ and $\frac{a_2}{1+\frac{a_1}{b_1}}$ respectively. This is illustrated in Figure 3.

Likewise from L we can see that the unit shock in x_3 will have full impact, $\frac{a_2}{a_1}$, on x_1, while no effect on x_2, for $b_1 \to 0$, (remembering that the horizontal shift in the demand curve is the vertical shift, a_2, multiplied by the numerical inverse slope, $\frac{1}{a_1}$). That is, when the supply curve is completely flat, demand shocks will have full effect on quantity, while no effect on prices. A corresponding argument goes for $a_1 \to 0$, a horizontal demand curve.

Thus, as is well known, the impact of, say, demand and supply shifts is completely determined by the slopes (partial derivatives) of the curves. This is exactly what the loadings matrix captures.

Completely in line with this we have the result that from the C matrix we can read off the comparative static effects. So the long-run impact of demand and supply shock are, naturally, the comparative static effects from our simple economic theory model. For example, the partial derivative of the equilibrium price of chicken, x^*_{2t}, with respect to the price of other meat, x_{3t}, $\frac{\partial x^*_{2t}}{\partial x_{3t}}$, computed from (12), is $\frac{b_1 a_2}{a_1+b_1} = \frac{a_2}{1+\frac{a_1}{b_1}}$, i.e. the second element in the first column of L, third column of C.

Note, from the C matrix in (23) we can see that the shocks to x_1 and x_2 have no permanent effect on any of the variables, since the two first columns contain zeros only. As mentioned, this corresponds to the fact that these shocks do not affect the positions of any of the curves, in contrast to the shocks ε_{3t} and ε_{4t}.

Before continuing, for further illustration, we can consider a case where a shock hits both curves. For example, in the above example, we might have that a rise in the price of other meat, x_3, also affects chicken supply decisions, say, by shifting the supply curve upwards. This means that the model is now

$$\begin{aligned} x^d_{2t} &= a_0 - a_1 x_{1t} + a_2 x_{3t} \\ x^s_{2t} &= b_0 + b_1 x_{1t} + b_2 x_{4t} + b_3 x_{3t} \end{aligned} \tag{24}$$

so that x_3 enters in the supply-relation with a coefficient b_3. The common trends are, of course, still the same as before, but now the loadings matrix is

$$L = \begin{pmatrix} \frac{a_2-b_3}{a_1+b_1} & -\frac{b_2}{a_1+b_1} \\ \frac{a_2 b_1+a_1 b_3}{a_1+b_1} & \frac{a_1 b_2}{a_1+b_1} \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{25}$$

Since both curves are shifted upwards, the effect on the equilibrium price of chicken, x^*_2, is unambiguously positive, while the effect on the equilibrium quantity of chicken, x^*_1, is of course only positive if the demand curve is shifted more than the supply curve, that is if $a_2 > b_3$, as is completely clear from the first element of L in (25). As is well known, note that the supply relation is not identified in this case, so we would need further variables or restrictions to obtain identification. The demand relation is still identified, since $b_2 > 0$.

Another thing one can note, is that the economic equilibrium in (12), call it, x^*_t, is a point on the attractor set as expected from the intuitive interpretation of the economic cross with the shifting curves above. As mentioned, the attractor set in this case can be written as $A \equiv \{x \in R^4 \mid \beta' x = -s\}$ remembering $s = -(a_0, b_0)'$. It is easy to show that $x_t^* = Cx_t - \alpha(\beta'\alpha)^{-1}s$, which is what corresponds to a *long-run value*, defined as $x_{\infty,t} \equiv \lim_{h\to\infty} E[x_{t+h} \mid x_t]$ (Johansen 2005). From this, it is seen that $\beta' x_t^* = -s$, i.e. $x_t^* \in A$. We note that it is exactly the exogenous variables x_3 and x_4 that determine the position of the long-run value on the the attractor set at time t, corresponding to where in Figure 1 the curves cross.

Finally, we also note that the type of cointegration that this typical kind of theory model implies is what is called an *irreducible* cointegration relation, i.e. stationarity is lost if we remove only one variables.

4.3 Transitory dynamics

Above we have considered the long-run or equilibrium movements, which is what economic theory models often are about. If adjustment in markets is relatively rapid, as "Classical-orientated" economists are more likely to claim than "Keynesian-orientated" economists, the equilibrium assumption made in economic models seems empirically reasonable in a sense, and it might be justified to focus on equilibrium behavior solely. However, if it is slow we have to take the process of adjustment seriously - perhaps more seriously than equilibrium movements. Obviously, it is practically irrelevant to speak about efficiency of a market equilibrium, if the market stays out of equilibrium for very long periods implying *inefficient* allocations.

As is well known, among macroeconomists, one of the great controversies is the question of how long the "short run" really is - say, how long is the period with increasing output, decreasing unemployment in the wake of a positive shock to aggregate demand, before inflation picks up and crowds out good times via a higher real exchange rate, for example. Obviously, this controversy implies a corresponding disagreement in terms of policy recommendations about stimulating employment via aggregate demand, say.

Thus, it seems clear that disequilibrium or transitory dynamics are extremely important and interesting to gain empirical insight into.

As mentioned, the behavior of the economy outside equilibrium is not modelled in the simple type of static theory models considered in this paper. Disequilibrium behavior can be complex and take many different forms, depending on the particular case. For example, whether we have rationing conditions, whether the physical properties of the good in question allow it to be kept in stock or not, costs in relation to changing prices, the informational structure of the market in question and so on, and so on.

In the CVAR, by postulating that agents are trying to realize their contingent plans once these have been violated by an unanticipated shock, we have in fact allowed for a wealth of possibilities in terms of disequilibrium dynamics (adjustment to long-run relations). The general expression for the transitory dynamics of adjustment is captured by the matrix, C_h^*, which in our case, the VAR(1), is $C_h^* = \alpha(\beta'\alpha)^{-1}(I_r + \beta'\alpha)^h\beta'$. How this evolves under the process of adjustment, i.e. for h growing, depends on the largest of the eigenvalues of $(I_r + \beta'\alpha)$, that is the largest of the "stationary roots of the process". In terms of Figure 3 we can say, that in the hypothetical absence of other shocks than the one in x_3 illustrated in the figure, $C + C_h^*$ gives us the exact position of the economy in the diagram after h periods, corresponding to the Impulse Response function in (7).

As in the underlying spirit of the present paper, let us consider a simple example of transitory dynamics that could be economically relevant. In particular, we return to the example above where we consider the effect of a unit change in the price of other meat. Figure 3 showed the long-run effect of this, and the purpose is now to trace out the path from the initial equilibrium position, x_t^*, to the new equilibrium.

To simplify, let us first consider a case where the price does not adjust directly when demand shifts. Rather, consumers buy more chicken at the given price because x_3 has risen - pork and beef have become expensive (the substitution effect dominates). Realistically, we assume that stocks are possible but limited. As producers see their stocks fall, in the wake of a positive demand shift, they start producing more, but at higher marginal costs so that price has to be raised.

Thus, our model is simply (20) with $\alpha_2 = \alpha_3 = 0$, i.e.

$$\begin{aligned}
\Delta x_{1t} &= -\alpha_1 a_1 (x_1 - (\frac{a_0}{a_1} - \frac{1}{a_1} x_2 + \frac{a_2}{a_1} x_3))_{t-1} + \varepsilon_{1t} \qquad (26) \\
\Delta x_{2t} &= -\alpha_4 (x_2 - (b_0 + b_1 x_1 + b_2 x_4))_{t-1} + \varepsilon_{2t} \\
\Delta x_{3t} &= \varepsilon_{3t} \\
\Delta x_{4t} &= \varepsilon_{4t}
\end{aligned}$$

where we have normalized on x_1 in the demand relation which seems natural.

Let us assume that producers face some amount of menu costs each, and that price setting is *staggered*, so that the average price level, x_2, is likely to change only *gradually* in the wake of demand shocks, say. Thus, even though the average actual price, x_2, is below its optimal level determined by the supply curve, it is probably rising rather slowly to begin with. In terms of the CVAR in (26), this could correspond to a small value of α_4. Some paths of the adjustment following a unit shock in x_3 are illustrated in Figure 4. This is just another way of illustrating the impulse response function.

Path P_a corresponds to the staggering case (a small α_4) of rigid prices, whereas in the case of P_b prices are less rigid (a higher α_4). All paths have been computed from simulating (26) with specific values $\alpha_1 = 0.1$, $a_0 = 12$, $a_1 = 2$, $a_2 = 2$, $b_0 = 1$, $b_1 = 1$, $b_2 = 1$. P_a has $\alpha_4 = 0.1$ and P_b has $\alpha_4 = 1.2$. As can be seen from the dots on the paths, the bulk of adjustment in prices, around 80%, has been eliminated after only 5 periods when $\alpha_4 = 1.2$, whereas 80% takes around 15 periods when $\alpha_4 = 0.1$. Alternatively, after 5 periods, only around 20 percent has been eliminated in the "staggered" case with $\alpha_4 = 0.1$.

For some goods stocks are physically impossible, for example many services. In such cases sales are equal to production or supply. This means that when demand shifts, say as a result of a unit rise in x_3, adjustment has to take place along the supply curve. Such case can of course also be modelled. Path P_c illustrates this. Note that, now we have introduced α_2 again, even

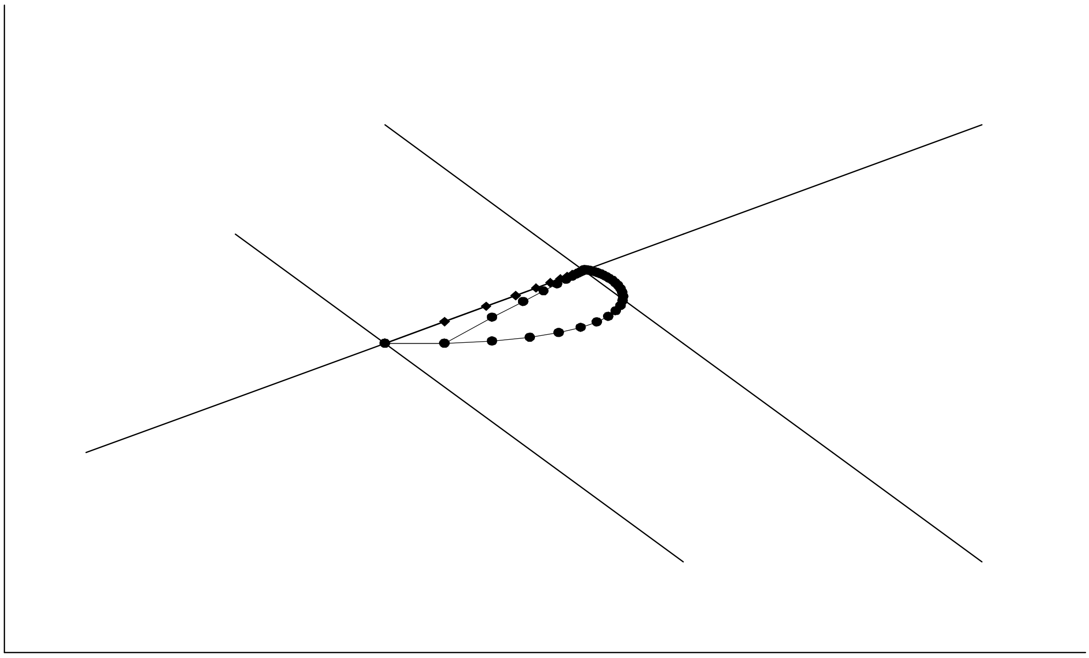

Figure 4: Three cases of transitory dynamics of adjustment in the wake of a demand shock: P_a is the curvy one (lowest) and shows slow adjustment in the price level. P_b shows much faster price adjustment. The last case, P_c, is adjustment along the supply curve.

though we could get adjustment along the supply curve with $\alpha_2 = 0$, having a very small α_1 and $\alpha_4 = 1$, say. The process of adjustment would be rather slow however.

So the model is now

$$\begin{aligned}
\Delta x_{1t} &= -\alpha_1(x_2 - (a_0 - a_1 x_1 + a_2 x_3))_{t-1} + \varepsilon_{1t} \\
\Delta x_{2t} &= -\alpha_2(x_2 - (a_0 - a_1 x_1 + a_2 x_3))_{t-1} - \alpha_4(x_2 - (b_0 + b_1 x_1 + b_2 x_4))_{t-1} + \varepsilon_{2t} \\
\Delta x_{3t} &= \varepsilon_{3t} \\
\Delta x_{4t} &= \varepsilon_{4t}
\end{aligned} \tag{27}$$

With $\alpha_2 = \alpha_1$ we get adjustment along the supply curve. Note that since adjustment is along the supply curve the value of α_4 is irrelevant in this case, as supply plans are always realized. The simulation was done for $\alpha_2 = \alpha_1 = 0.1$.

While considering this example, there is one particularly interesting case of adjustment along the supply curve. This is the case where all adjustment in quantity and prices takes place in the following period. This is can also be referred to as full adjustment (instantaneous adjustment) to unanticipated shocks, and essentially describe *"Rational Expectations"* or *"Model Consistent Expectations", given an unexpected shock* (see below). Agents simply calculate the new equilibrium given the unit shock to x_3, and adjust their behavior accordingly.

To model this case, we can proceed by asking the question: Given the long-run parameters, i.e. the $a_i s$ and $b_i s$, what should the short-run adjustment parameters, the αs, be, in order for the adjustment in period $t+1$, to the unanticipated unit shock to x_3 in period t, to be exactly the (full) long-run effect? From our C matrix we have the long-run effects, and from forwarding (26) by one period we can compute the changes in x_1 and x_2 from period t to period $t+1$. Assuming that we are in equilibrium to begin with (period $t-1$) we get the following two simple conditions

$$\begin{aligned} \Delta x_{1t+1} &= \alpha_1 a_2 = \frac{a_2}{a_1+b_1} \Leftrightarrow \alpha_1 = \frac{1}{a_1+b_1} \\ \Delta x_{2t+1} &= \alpha_2 a_2 = \frac{b_1 a_2}{a_1+b_1} \Leftrightarrow \alpha_2 = \frac{b_1}{a_1+b_1} \end{aligned} \tag{28}$$

Thus, this kind of hypothesis, involves cross restrictions between long-run and short-run parameters.

We can take this further by also letting the adjustment to supply shocks be instantaneous. We then have to reintroduce α_3 in the model as well. So we are back at the model in (20). It is easy to show that instantaneous adjustment to supply shocks, ε_4, implies $\alpha_3 = \frac{1}{a_1+b_1}$ and $\alpha_4 = \frac{a_1}{a_1+b_1}$. Thus, the adjustment matrix is given by

$$\alpha = \begin{pmatrix} -\frac{1}{a_1+b_1} & \frac{1}{a_1+b_1} \\ -\frac{b_1}{a_1+b_1} & -\frac{a_1}{a_1+b_1} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{29}$$

while the rest of the model is the same as before. Note that since $\alpha_\perp$, β and $\beta_\perp$ are unchanged the I(1) condition is still fulfilled. Since adjustment is instantaneous, we expect the impulse response function to equal C. Computing $(I_r + \beta'\alpha)$ with α given by (29) gives $(I_r + \beta'\alpha) = 0$ since $\beta'\alpha = -I_r$. Thus the transitory effect in the impulse response function, $C_h^* = \alpha(\beta'\alpha)^{-1}(I_r + \beta'\alpha)^h\beta'$, is a zero matrix as intuitively expected (See (7) and (6)).

Therefore, if such conditions are fulfilled, this corresponds to full adjustment to unanticipated demand and supply shocks in the following period. Note that, this corresponds to the equilibrium error process being a white noise.

As mentioned, this can be interpreted as what economists usually call "Rational Expectations" or "Model Consistent Expectations", when the shocks are unexpected (See for example Heijdra and van der Ploeg 2002, Chapter 3). This is the type of expectations originating from the ideas of Muth (1961). The obvious and of course still deliberately stylized application is to consider the AS-AD model with these restrictions imposed, combined with a vertical long-run (classical) AS curve, implying ineffectiveness of economic policy (See Section 4.4). For an example like that, see Heijdra and van der Ploeg (2002).

Note that, the shocks are unexpected (unanticipated) here. If they were expected, instantaneous adjustment would take place in the same period as the shock occurred (the static contingent plans are optimal) and we would jump from one equilibrium to the next in period t.

Extreme hypotheses like "Rational Expectations" are of course likely to reject but nevertheless very interesting to formulate and test in the statistical model. Note that, this hypothesis can be formulated in a more general CVAR, that, in principle, has nothing to do with the theoretical model above. The idea is simply to impose the restriction that all adjustment takes place immediately in the period after the shock. Finally, in passing, note that if the equilibrium is efficient, as is the competitive equilibrium, this is a test for efficient markets exposed to unforeseen shocks.

Thus, to sum up, these simple examples demonstrate how central and controversial hypotheses about, staggering price setting, rational model based expectations are translated to restrictions on the CVAR, in a straightforward manner. Finally, we note that the CVAR also allows adjustment paths that are spiralling, i.e. smooth harmonic oscillations in x_1 and x_2, kinked oscillations, full adjustment to the demand relation etc. etc. Thus, even this simple system of first order difference equations is an extremely flexible model nesting many special cases of economic interest.

4.4 Adding lags - A static AS-AD example

In the VAR model above we have only one lag. That is, $x_t = \Pi_1 x_{t-1} + \varepsilon_t$, is the model. On the one hand, adding one more lag to this model complicates the analysis a bit. On the other we get a more flexible model, and in this section we look at an example where this extra flexibility seems useful from an economic point of view.

Consider the static Aggregate Supply - Aggregate Demand (AS-AD) model of prices and output (GDP) for the closed economy. In traditional macroeconomic theory it seems consensus that short-run fluctuations in output (business cycles) are mainly due to shifts in aggregate demand (combined with nominal rigidities, say, and the presence of idle resources), while long-run growth is determined by supply factors (production inputs) primarily (Blanchard 1997 and Solow 1997). In other words, shifts to the AD-curve seem to influence output only for a limited period of time, and not in the long run: Eventually, the aggregate price level takes the full adjustment. For instance, if demand is shifted by an a% increase in money supply, this produces an a% shift in prices in the long run. Thus, when analyzing the AS-AD model of prices, output and some determinant of aggregate demand, say, money supply, traditional macroeconomics would let the endogenous/exogenous division depend on the time horizon of the study: The (very) short-run AS-AD model would have prices as exogenous (the IS-LM-model) and output (and interest rates) as endogenous, while in an AS-AD model of the long run, this is reversed so that output is exogenous and prices endogenous. Whereas this, to some extent, represents the consensus way of analyzing the short and the long run, there is far from agreement concerning the question of how long this "limited period of time" really is, as mentioned.

By adding only one more lag to the VAR, we can actually formulate an AS-AD model that has the property that demand shifts have (potentially large) effects on output in the short run and small effects on prices, while no effect on output and full effect on prices in the long run. Moreover, as expected from the examples above, we are able to model the full path of transitory dynamics, enabling us to answer the central question "How long is the short run?".

Consider the system $x_t' = (p_t, y_t, m_t)$, where p is the aggregate price level, y is output and

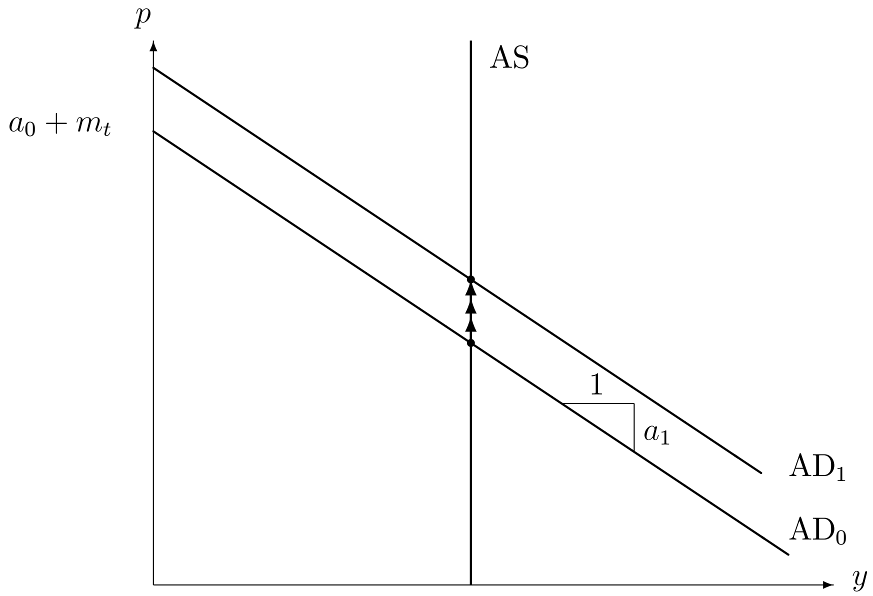

Figure 5: The Classical AS-AD model.

m is money supply. The (inverse) static Aggregate Demand curve is given by the relation

$$p_t = a_0 - a_1 y_t + a_2 m_t \tag{30}$$

First, look at the system in the long run - call it the "Classical AS-AD model". This would correspond to the following CVAR

$$\begin{aligned} \Delta p_t &= -\alpha_1 (p_{t-1} - (a_0 - a_1 y_{t-1} + a_2 m_{t-1})) + \varepsilon_{p,t} \\ \Delta y_t &= \varepsilon_{y,t} \\ \Delta m_t &= \varepsilon_{m,t} \end{aligned} \tag{31}$$

and with the "classical" homogeneity restriction $a_2 = 1$. The model is illustrated in Figure 5, for a monetary expansion (a positive shift in m). This simple model with one lag only does not allow for other than vertical adjustment (adjustment in prices only) as is illustrated. The long-run effects are given by the C matrix

$$C = \begin{pmatrix} 0 & -a_1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{32}$$

Which now should be relatively obvious from inspecting Figure 5. Note that, instantaneous adjustment to unanticipated shocks ("Rational" or "Model based" expectations) in this simple model is of course the case $\alpha_1 = 1$, corresponding to $I_r + \beta'\alpha = 1 - \alpha_1 = 0$. Note that, this was actually assumed in the preliminary example in Section 4.1.

In order to allow for the possibility that shifts in money supply affects output in the short run through aggregate demand[10], we introduce three new parameters, γ_1, γ_2 and γ_3, to get the model

$$
\begin{aligned}
\Delta p_t &= -\alpha_1(p_{t-1} - (a_0 - a_1 y_{t-1} + m_{t-1})) + \varepsilon_{p,t} \\
\Delta y_t &= -\gamma_1 \Delta p_{t-1} + \gamma_2 \Delta y_{t-1} + \gamma_3 \Delta m_{t-1} + \varepsilon_{y,t} \\
\Delta m_t &= \varepsilon_{m,t}
\end{aligned} \tag{33}
$$

where all parameters are positive as usual. From (33) we can see that there are at least transitory effects from changes in money supply on output. As mentioned, economic theory would say that even though these transitory effects are present, and perhaps very persistent, there is no long-run impact on output, only on prices. Thus, the C-matrix should be the same as in (32).

The C-matrix for the model in (33) is

$$
C = \begin{pmatrix} 0 & \frac{a_1}{\gamma_2+\gamma_1 a_1 - 1} & 1 + \frac{a_1(\gamma_3-\gamma_1)}{\gamma_2+\gamma_1 a_1-1} \\ 0 & \frac{-1}{\gamma_2+\gamma_1 a_1-1} & \frac{-(\gamma_3-\gamma_1)}{\gamma_2+\gamma_1 a_1-1} \\ 0 & 0 & 1 \end{pmatrix} \tag{34}
$$

remembering that $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$ and $\Gamma = I - \Gamma_1$ in this case. The I(1) condition is $\gamma_2 + \gamma_1 a_1 \neq 1$.

For the C-matrix in (34) to equal that in (32) we need the restrictions

$$
\gamma_3 = \gamma_1 \text{ and } \gamma_2 + \gamma_1 a_1 = 0 \tag{35}
$$

Where we note that the last condition insures I(1), provided that the roots of the characteristic polynomial fulfill (3).

Note that in the model in (31), both m and y are strongly exogenous, while in (33), m is still strongly exogenous, but y is only weakly exogenous. Loosely speaking, in the words of economists we could say that y is only exogenous in the long run.

Thus, the model in (33) with (35) imposed is an AS-AD model with Classical long-run properties and Keynesian short-run properties - i.e. what could be called the "Neoclassical Synthetic AS-AD model" (Blanchard 1997, Heijdra and van der Ploeg 2002). Such model can be estimated, and if successful, give answers to how long the short-run is, for how long aggregate demand policy stimulates production etc..

This model is illustrated in Figure 6, where some hypothetical path of adjustment to a monetary shock, $\varepsilon_{m,t} = 1$, is illustrated.

In this AS-AD model, the hypothesis of Rational Expectations, corresponds to the restrictions

$$
\alpha_1 = 1 \text{ and } \gamma_3 = 0 \tag{36}
$$

[10] According to theory, money supply affects real money balances when prices are rigid and hence interest rates and thereby aggregate demand (investment and consumption demand).

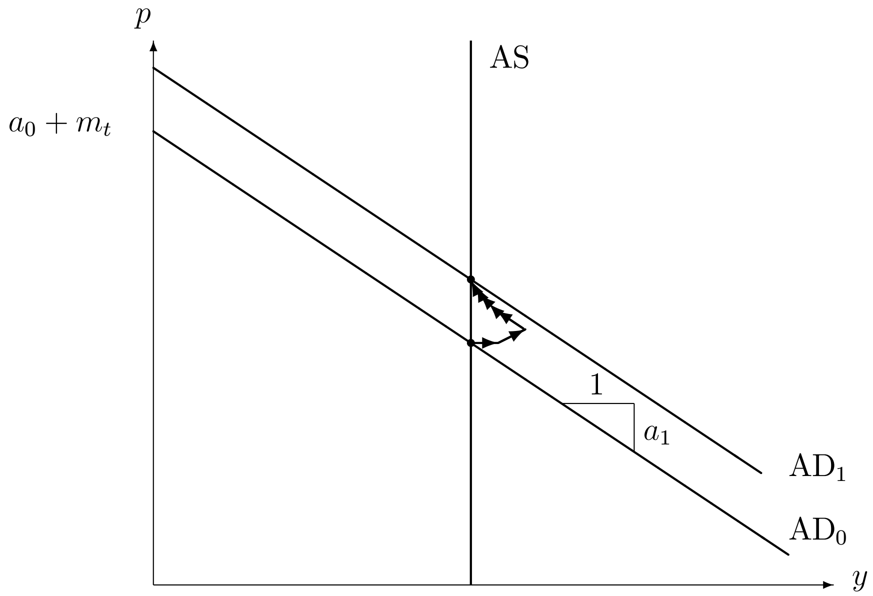

Figure 6: The AS-AD model with Classical long run properties and Keynesian short run properties.

In terms of the graph in Figure 6, if these restrictions are fulfilled, a shift in the AD curve at time t will make the economy jump to the new equilibrium in period $t+1$, that is, there is no effect on output from a monetary expansion, only the price level will rise. Thus, this example essentially captures how economic policy becomes ineffective in the presence of Rational Expectations, as argued by Lucas and Sargant in the 1970s (See for example Blanchard 2000).

4.5 The general equilibrium extension

Static theory models like the one above, are of course too simple and abstract in many respects. There are many crucial extensions that result in theory models that are much closer to what many modern applied economists work with. We shall briefly discuss such extensions in Section 6. However, if we view the model from above as a partial equilibrium model, as we have done so far (except for the AS-AD case), one immediate extension is obvious, namely that of imposing *general equilibrium.*

The present section considers a simple general equilibrium extension of the model from Section 3. The purpose of this extension is partly to see how a simple general equilibrium model looks in terms of the CVAR, and hence be a bit more general than above, partly to address the question as to when the ceteris paribus assumption underlying the partial equilibrium analysis is appropriate. As we shall see below, basically all insight from above generalizes completely.

The partial equilibrium model assumes that the price of other meat, x_3, is exogenous, and hence, does not allow for the endogenous chicken price, x_2, to feed back on x_3, which seems unrealistic: A high (equilibrium) price of chicken, caused by a rise in x_3, would probably raise the demand for other meat, raising its price, x_3, which, in turn, would feed back positively on chicken demand so the increase in chicken price would be reinforced, and so on, and so on. In other words we would need to impose general equilibrium and also include the market for other

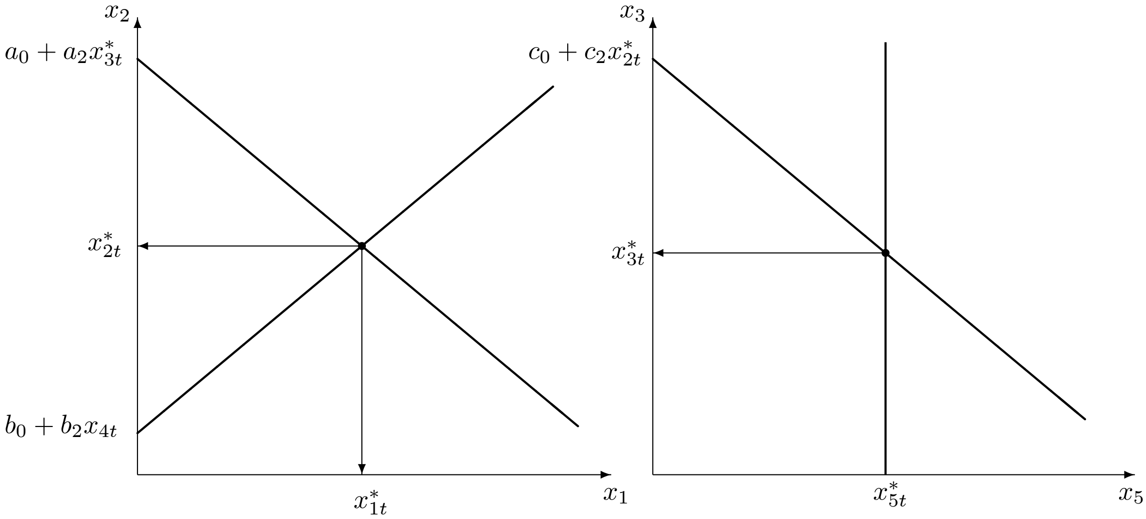

Figure 7: The general equilibrium model. Note how the demand curves are drawn for the equilibrium values of the price on the related market. Hence, a shift in one market starts a sequence of shifts in both markets.

meat[11]. As with our simple static economic model above, such consideration is well-known in economic theory. The purpose here, is to see how they can be related to the CVAR.

To illustrate the basic insights as simple as possible, we extend the model of the chicken market by the market for other meat, call it x_5, and, in particular, assume that the supply of other meat is exogenous. Demand for other meat is of course related not only to the price of other meat, x_3, but also to the price of chicken, x_2.

Thus, our new two-market general equilibrium model looks like

$$\begin{aligned} x_{2t} &= a_0 - a_1 x_{1t} + a_2 x_{3t} \\ x_{2t} &= b_0 + b_1 x_{1t} + b_2 x_{4t} \\ x_{3t} &= c_0 - c_1 x_{5t} + c_2 x_{2t} \\ & x_{5t} \text{ exogenous} \end{aligned} \tag{37}$$

Solving the model gives the general equilibrium

$$\begin{aligned} x^*_{1t} &= \frac{a_0 + a_2(c_0 - c_1 x_{5t}) + (a_2 c_2 - 1)(b_0 - b_2 x_{4t})}{D} \\ x^*_{2t} &= \frac{(b_0 - b_2 x_{4t})a_1 + b_1(a_0 + a_2(c_0 - c_1 x_{5t}))}{D} \qquad D \neq 0 \\ x^*_{3t} &= \frac{(b_1 + a_1)(c_0 - c_1 x_{5t}) + c_2((b_0 - b_2 x_{4t})a_1 + b_1 a_0)}{D} \end{aligned} \tag{38}$$

where $D \equiv a_1 - b_1(a_2 c_2 - 1)$ is the determinant of the coefficient matrix to the system. Thus, existence of equilibrium is given if and only if $D \neq 0$, which is assumed. The model is illustrated in Figure 7, for given values of the exogenous variables, x_4 and x_5.

[11]In practice, "General Equilibrium" analysis, implies the imposition of equilibrium on, not all markets, but only one or a few *related* markets.

Let us now relate the extended model to the CVAR. To keep it simple we shall introduce only one additional adjustment coefficient, in the α matrix, α_5. As we have seen, this simplification will not affect the common trends, and hence, the long-run behavior, which is the focus of this section, only the transitory dynamics are affected. Thus, equationwise the ECM can be written

$$\begin{aligned}
\Delta x_{1t} &= -\alpha_1(x_2-(a_0-a_1x_1+a_2x_3))_{t-1}+\alpha_3(x_2-(b_0+b_1x_1+b_2x_4))_{t-1}+\varepsilon_{1t}\\
\Delta x_{2t} &= -\alpha_2(x_2-(a_0-a_1x_1+a_2x_3))_{t-1}-\alpha_4(x_2-(b_0+b_1x_1+b_2x_4))_{t-1}+\varepsilon_{2t}\\
\Delta x_{3t} &= -\alpha_5(x_3-(c_0-c_1x_5+c_2x_2))_{t-1}+\varepsilon_{3t}\\
\Delta x_{4t} &= \varepsilon_{4t}\\
\Delta x_{5t} &= \varepsilon_{5t}
\end{aligned} \tag{39}$$

This corresponds to

$$\alpha=\begin{pmatrix} -\alpha_1 & \alpha_3 & 0\\ -\alpha_2 & -\alpha_4 & 0\\ 0 & 0 & -\alpha_5\\ 0&0&0\\ 0&0&0\end{pmatrix},\ \beta=\begin{pmatrix} a_1 & -b_1 & 0\\ 1 & 1 & -c_2\\ -a_2 & 0 & 1\\ 0 & -b_2 & 0\\ 0&0&c_1\end{pmatrix} \text{ and } \phi=\alpha s,\ s=\begin{pmatrix}-a_0\\ -b_0\\ -c_0\end{pmatrix} \tag{40}$$

with orthogonal complements

$$\alpha_\perp=\begin{pmatrix}0&0\\0&0\\0&0\\1&0\\0&1\end{pmatrix},\ \beta_\perp=\begin{pmatrix}\frac{(a_2c_2-1)b_2}{D} & \frac{-a_2c_1}{D}\\ \frac{a_1b_2}{D} & \frac{-b_1a_2c_1}{D}\\ \frac{a_1c_2b_2}{D} & \frac{-(a_1+b_1)c_1}{D}\\ 1&0\\0&1\end{pmatrix} \tag{41}$$

and common trends and their loadings

$$\mathrm{CT}=\begin{pmatrix}\Sigma_{i=1}^t\varepsilon_{4i}\\ \Sigma_{i=1}^t\varepsilon_{5i}\end{pmatrix},\ L=\begin{pmatrix}\frac{(a_2c_2-1)b_2}{D} & \frac{-a_2c_1}{D}\\ \frac{a_1b_2}{D} & \frac{-b_1a_2c_1}{D}\\ \frac{a_1c_2b_2}{D} & \frac{-(a_1+b_1)c_1}{D}\\ 1&0\\0&1\end{pmatrix} \tag{42}$$

resulting in the long-run matrix

$$C=\begin{pmatrix}0&0&0&\frac{(a_2c_2-1)b_2}{D} & \frac{-a_2c_1}{D}\\ 0&0&0&\frac{a_1b_2}{D} & \frac{-b_1a_2c_1}{D}\\ 0&0&0&\frac{a_1c_2b_2}{D} & \frac{-(a_1+b_1)c_1}{D}\\ 0&0&0&1&0\\0&0&0&0&1\end{pmatrix} \tag{43}$$

As can be seen, the interpretation from the partial equilibrium model generalizes completely, in that, the theoretically exogenous variables are the common trends, the loadings matrix captures how the slope and partial derivatives of the curves determine their impact on the variables, and the C matrix shows the comparative static effects. As we now consider two markets it is slightly more complicated, however - but yet still relatively simple. Since the basic intuition is the same for all elements in the C matrix, instead of an element-wise interpretation, we shall focus on the interpretation of the effect of a unit rise in x_5 (the exogenous supply of other meat) on the traded quantity of chicken, x_1, and be thorough:

The typical "textbook like" interpretation of the general equilibrium comparative static effect, $\frac{\partial x_{1t}^*}{\partial x_{5t}}$, i.e. the fifth element of the first row of the C matrix, could be as a sequential interaction between the two markets with market clearing in each market, each round. As before, in static equilibrium theory models like this, we abstract from the dynamics of the interaction altogether, and just consider the economy after all adjustment has taken place, whereas such interaction is fully modelled in the CVAR (In general, with no market clearing in each round of course). Let us now consider each "round":

Round 1: A unit rise in x_5 leads to a fall in x_3 by c_1 units. Round 2: The fall in the price of other meat, x_3, induces a downward shift in the chicken demand curve by c_1a_2 units. This, in turn, reduces the equilibrium value of x_1 by $\frac{c_1a_2}{b_1+a_1}$ units, and the equilibrium price of chicken x_2 by $\frac{b_1c_1a_2}{b_1+a_1}$. Round 3: The fall in x_2 then feeds back on the market for other meat, by shifting the initial demand curve downwards by $\frac{c_2b_1c_1a_2}{b_1+a_1}$ units. Since the quantity of other meat is exogenous, x_5, i.e. the supply curve is vertical, this means that equilibrium price of other meat falls by exactly this amount. Round 4: As expected this fall in x_3 spills over to the chicken market, by shifting the chicken demand curve downwards again by $\frac{c_2b_1c_1a_2^2}{b_1+a_1}$ units which reduces the equilibrium value of x_1 by $\frac{c_2b_1c_1a_2^2}{(b_1+a_1)^2}$ units, and the equilibrium price of chicken x_2 by $\frac{a_2^2b_1^2c_2c_1}{(b_1+a_1)^2}$ units. Round 5: Again the fall in x_2, and the fact that x_5 is exogenous implies yet another reduction of the equilibrium value of x_3 by $\frac{a_2^2b_1^2c_2^2c_1}{(b_1+a_1)^2}$. Round 6: Again the spillover to the chicken market shifts chicken demand and implies a reduction in x_1 by $\frac{a_2^3b_1^2c_2^2c_1}{(b_1+a_1)^3}$ units, and the equilibrium price of chicken x_2 by $\frac{a_2^3b_1^3c_2^2c_1}{(b_1+a_1)^3}$ units. Round 7 implies a reduction in x_3 by $\frac{a_2^3b_1^2c_2^3c_1}{(b_1+a_1)^3}$ units, and the story goes on.

Summing all the falls in x_1 from each round gives us the series

$$V = \frac{c_1a_2}{b_1+a_1} + \frac{a_2^2b_1c_2c_1}{(b_1+a_1)^2} + \frac{a_2^3b_1^2c_2^2c_1}{(b_1+a_1)^3} + \frac{a_2^4b_1^3c_2^3c_1}{(b_1+a_1)^4} + ... \tag{44}$$

Defining a new series as $W \equiv V\frac{c_2b_1}{c_1}$ we get

$$W = \frac{a_2c_2b_1}{b_1+a_1} + \left(\frac{a_2c_2b_1}{b_1+a_1}\right)^2 + \left(\frac{a_2c_2b_1}{b_1+a_1}\right)^3 + \left(\frac{a_2c_2b_1}{b_1+a_1}\right)^4 + ... \tag{45}$$

which is convergent if $|s| < 1$ where $s \equiv \frac{a_2c_2b_1}{b_1+a_1}$, implying that V is also convergent for $|s| < 1$. It is intuitively clear that if the model is to have a stable economic equilibrium the feedback effects between the markets must be numerically smaller and smaller in every round. This is of course exactly the requirement that $|s| < 1$. Note that $s > 0$ as all parameters are positive by assumption, and $s \lesseqqgtr 1 \Leftrightarrow D \gtreqqless 0$. For $D = 0$ $(s = 1)$ existence is lost and for $D < 0$ $(s > 1)$ the equilibrium is unstable, while $D > 0$ $(s < 1)$ implies a stable equilibrium with partial derivatives in (38) with qualitatively reasonable signs. So, the latter is assumed.

When $s < 1$, V, the total fall in x_1^* resulting from a unit rise in x_5, can be computed from

$$\begin{aligned} W &= \frac{s}{1-s} = \frac{a_2 c_2 b_1}{D} \text{ and } V = \frac{W c_1}{c_2 b_1} \Leftrightarrow \\ V &= \frac{a_2 c_1}{D} \end{aligned} \tag{46}$$

thus the numerical comparative static effect.

As is well known, the comparative statics of general equilibrium models are often quantitatively but can also be qualitatively different relative to the partial equilibrium model. As can be seen by comparing the C matrices in the partial and the general equilibrium model, (23) and (43) respectively, the effect of a rise in x_4 on x_1, i.e. from a shift in the supply curve, can be qualitatively different. The partial equilibrium effect is unambiguously negative, $-\frac{b_2}{a_1+b_1}$, while in general equilibrium the effect is $\frac{(a_2c_2-1)b_2}{D}$, which is negative only if $a_2c_2 < 1$. In terms of the graphs, the partial equilibrium model shows the initial upward shift in the chicken supply curve, and then the story ends. In the general equilibrium model the resulting rise in the price of chicken, x_2, spills over to the market for other meat, and shifts up the demand curve on this market, which in turn feeds back and shifts chicken demand upwards etc., etc. So, in the wake of the shift in the supply curve, there is a sequence of upward shifts in the chicken demand curve, and if the sum of these shifts is greater than the supply shift, x_1^* will rise. This is the case when $a_2c_2 > 1$, as this describes a situation where the interaction between the two prices are sufficiently strong to dominate the initial partial effect.

Note that, the general equilibrium effects on x_1 and x_2, from changes in x_4, are equivalent to the partial equilibrium effects when c_2 is zero. In such case there is an influence from x_3 on x_2 but not the other way round. Loosely speaking, such case could correspond to a case of a small domestic market for chicken affected be a large international market for other meat.

Furthermore, if, in addition to $c_2 = 0$, we have $c_1 = 1$, the general equilibrium model basically coincides with the partial equilibrium model. In this case a unit fall in x_5 produces a unit rise in x_3 that has the partial equilibrium effect on the chicken market. In such case the stochastic trend in x_3 is just the common trend $\Sigma_{i=1}^t \varepsilon_{5i}$ with negative sign. If this were the case economists would rather prefer the partial equilibrium model since it is easier to handle. Likewise, since it implies fewer parameters this would also be preferred from an econometric point of view. However, if it is *not* the case, we have seen that comparative statics change, even qualitatively, and so the general equilibrium extension is crucial.

Thus, if we formulate the general equilibrium initially we can test whether the partial model is valid, by the parameter restrictions $c_2 = 0$ and $c_1 = 1$. Alternatively, one can start with the small system, and include the variables dictated by the partial equilibrium model. Then, one can test the exogeneity of x_3. If accepted, one can stick to the partial analysis, in principle, since its comparative statics would be the same as in the larger model. If general equilibrium interaction effects are important, this is likely to show up in the small model. For example, in practice, the test for weak exogeneity of x_3 would reject, and in addition to the chicken demand and supply cointegrating relations there would probably be a third relation between x_3 and x_2 that was (borderline) rejected as stationary, which would become stationary once the new variable, x_5, was included, to be interpreted as the demand relation for other meat.

There is a close correspondence between the general- and the partial equilibrium way of thinking, and the gradual model building approach that is advocated for in Juselius (1992) and Juselius (2006). That approach exploits the invariance property of cointegration with respect to extending the information set, so that one can start with a smaller, and hence more manageable system, and then gradually extend by one variable at the time, or alternatively combine it with another small system. Such approach can be of crucial importance in applications involving several variables, say 5-10 or even more, by facilitating the practical identification of the long-run (cointegration) system. In view of the simple models above we could imagine that a practical approach could be to start with a partial equilibrium system of one market, and then add variables that could be the relevant general equilibrium ones (i.e. x_5). One could also imagine that we could combine results from two partial equilibrium system - two markets. The general equilibrium approach also resembles the "Joint Modelling Approach" in Juselius and MacDonald (2000), where aggregate goods-, foreign exchange- and financial markets are analyzed jointly.

Therefore, the examples above illustrate how theory information and basic economic principles can be of great help in the gradual model building approach in practice: Finding a "strange" non-stationary relation, we can ask whether this becomes stationary and interpretable by including the crucial ceteris paribus variable(s) modelled in general equilibrium models (other prices).

5 Discussion

Above we have considered a few deliberately stylized and naive examples of static theory models. In no way we claim to have represented modern macroeconomic theory models. Nor, that we have illustrated the complete theoretical apparatus of the CVAR, of course. Instead, the purpose has been to clarify the connection between theory models and the CVAR, by considering a few simple examples in detail. It is my belief that such steps facilitate communication between economist and VAR econometricians. On the one hand, it is easier for economists to see what is going on, and therefore be able to suggest hypotheses of economic interest more precisely, as well as criticize existing applied cointegration analyses. On the other hand, (new) VAR modellers might find it easier to use theoretical information in their particular application, once some simple examples have been seen, from which one can elaborate (which certainly is the idea).

It is also important to emphasize that all the restrictions on the CVAR, which we have derived from the theory model, are statistically *testable* restrictions. The emphasis is on testable, in the sense that theory models should not be "imposed on to the data", rather, we should start by formulating the unrestricted VAR as our statistical model. This, then serves as the general framework within which we can test the hypotheses above - i.e. the simple theory model should be *nested* within it. If all the restrictions cannot be jointly rejected the theory model seems successful. But, as shown by the examples above, it takes a lot of restrictions to be accepted in order for this to be the case. Also, note how all the fundamental assumptions of economic theory actually are testable, such as *existence* of equilibrium, the *endogenous/exogenous division, partial/general equilibrium* distinction, *Rational Expectations* etc.. In terms of our simple example we first test the cointegration rank, which is basically to test whether there *is* an economic cross in the first place (existence of equilibrium). We can then test whether the theoretically exogenous variables correspond to the strongly exogenous variables, and various hypotheses about the partial derivatives in the form of hypotheses on the cointegrating coefficients.

The examples above are of course rather restrictive and hence motivating a discussion. Obviously, we can discuss the above examples in relation to reality, that is in relation to what is likely to be found when analyzing real world data. So, this is what is done first. Subsequently, we shall briefly mention the most obvious extensions of the theory model, which can be related to the CVAR in some future paper, say.

5.1 Practical problems

In a real world analysis things are unfortunately much more complicated, blurred and inconclusive than above. First of all, above we assumed that all variables are I(1), whereas in practice nominal variables are often better described by I(2) processes. If one is lucky some nominal-to-real transformation is accepted in a nominal I(2) model, and one can therefore analyze a *real* model that, at the most, has I(1) variables, without loosing information (Kongsted 2005). If this is not the case a full I(2) analysis has to be carried out, which is much more complicated. Secondly, often extraordinary and exogenous events happen that has large effects on the system. In estimation this usually produces some outlying residuals. A tough question is how to model such events with dummies. Is the shock innovative (influencing the system as the innovations) or additive, is it permanent or transitory? We have also disregarded much of the other innovational deterministics, such as restricted trends etc. The assumption of constant parameters also needs attention in practice. Note how these concepts can be interpreted in light of our simple exposition in this paper. For instance, large innovational shocks to the exogenous variables, simply corresponds to large (non-normal) shifts to the curves, a *restricted* trend in a static partial supply and demand model could describe steady shifts in the demand curve each period as a result of some deterministic income growth, say. Likewise an economic cross might describe the DGP, but slopes and intercepts, as well as adjustment coefficients may vary over time.

In the present paper we have also abstracted from non-zero off-diagonal correlations in the error covariance matrix. In the chicken example it seems reasonable that shocks are uncorrelated. For example, why should *unanticipated* shocks to the price of inputs in chicken production in period t be correlated with shocks to the price of chicken, or of other meat, in the same period? There is an emphasis on unanticipated though. In many situations it does not seem reasonable to treat shocks as unanticipated. For instance, looking at a demand curve, while shocks to prices of substitutes and/or complements might not be expected from the viewpoint of the individual consumers, it seems more realistic that changes in income are, to a larger extent, expected. After all, a worker's income is determined by working hours, which are fixed or chosen by him or herself, times the wage rate, whose level and discrete changes are fixed by negotiations. If the contingent plans dictate static relationships, as is the case above, say, *current* consumption depends on *current* income, and an income change in period t is expected, it seems reasonable that the consumer will change the level of consumption in period t - not before (since the static relationship is optimal) not after (since the static relationship is optimal and since the change is anticipated). In this sense, I believe that anticipated changes will show up as off diagonal correlations. Of course, as mentioned the adjustment to an expected change might imply costs, so that it is optimal to spread out adjustment over time instead.

5.2 Extending the theory models

It is clear that the simple static models considered here do not constitute a fair characterization of modern "state-of-the-art" macroeconomic theory models. As mentioned, they are pedagogical points of departure. Needless to say, there is a vast number of possible extensions of the static equilibrium theory, and we shall briefly mention some of the most fundamental ones here.

The first concerns expectations in the structural relations. For example, current demand for chicken is probably related not only to current prices and income but also to the expectation about the future value of such variables, say. As changes in the current value of some variable are likely to influence expectations about the future value of that variable, abstracting from such expectation formation or treating it as constant, by considering a simple static model like above, is likely to be crucial with regards to prediction, as well as normative and positive statements based on the model. One possibility to advance in this direction is to try to combine the ideas in Johansen and Swensen (1999) with our simple examples above.

Another immediate and profound extension of theory is of course to add dynamics. From dynamic models we are likely to extract detailed hypothesis about the dynamics of adjustment etc.

As the purpose of the present paper was to consider some simple examples of theory models in detail, even though these extensions are crucial, they are left open for a future paper.

6 Summary and concluding remarks

In an attempt to contribute to the emergence of a larger degree of integration of economic theory models and the approach of the Cointegrated Vector-Auto-Regressive model (CVAR), this paper has taken a few steps, by considering the implications of simple static theory models in terms of the CVAR. Hopefully, the paper has demonstrated the great potential that the CVAR has as an empirical means of analyzing economic data on the basis of economic theory models, in particular. The spirit has been to keep it as simple and transparent as possible by looking at simple "text book like" examples of static theory models and translate these into the restrictions and concepts of the CVAR. Having seen such simple example it becomes easier to advance and consider more up-to-date theory models, I believe.

We started by showing how the theoretical set up with contingent plans combined with unanticipated shocks coincides with the ECM reparameterization of the regression function. Then, we looked at the simple static partial equilibrium model, and thought of the chicken market to fix ideas. This was subsequently embedded in a VAR(1). The theoretical exogenous variables were interpreted as the strongly exogenous variables, and hence the common trends. How these affected the endogenous variables was shown to be completely determined by the loadings matrix, as this reflected partial derivatives and slopes of the demand and supply curves. The long-run C-matrix was shown to have the comparative static effects as its elements, as would be expected from the underlying idea of comparative statics. Shocks with permanent effects were those to the exogenous variables, as these changed the positions of the demand and supply curves, whereas shocks with transitory effects only, were those to the endogenous ones, not affecting the position of the curves. The economic equilibrium was on the attractor set, and just as it was the values of the exogenous variables that determined where in the demand supply diagram the curves cross, in an equivalent manner, these determined where on the attractor set the equilibrium (the long-run value) was. The kind of structural relations we looked at were interpreted as irreducible cointegration relations.

By embedding the static theory model in the dynamic empirical model we also showed how the CVAR allows us to model transitory- or off-equilibrium dynamics, something not modelled in the theory model per se, and hence, extremely interesting to gain empirical insight into. Some stylized examples showed how we can relate important economic concepts such as menu costs and staggered price setting generating sluggishness in the aggregate price level, to the adjustment parameters of the CVAR. A particularly interesting case was considered, namely when *full* adjustment to an unanticipated shock to demand and/or supply takes place in the following period. This was interpreted as "Model Consistent Expectations" or "Rational Expectations" in the presence of unanticipated shocks. The exact restrictions on the VAR-model were easily derived, and hence showed how we can formulate and test some of the most controversial hypotheses in macroeconomics.

A simple example of an AS-AD model was also translated to a VAR. By including an additional lag in the VAR and imposing some simple restrictions, we could get a model with "Classical properties" in the long run, and "Keynesian properties" in the short run. That is, an AS-AD model where shifts in Aggregate Demand (due to monetary policy, say) would have effect on output only in the short run but not in the long run. In the long run only higher prices resulted. The examples also implied how the CVAR allows us to answer questions such as for how long output is affected by demand shocks before inflation starts crowding out the expansion, i.e. "how long the short run is" stated boldly. Such questions lie at the heart of macroeconomic controversy[9], underscoring the potential contributions from making VAR analyses.

The Rational Expectations hypothesis combined with this AS-AD model demonstrated the "policy ineffectiveness implied by Rational Expectations" as put forward by Lucas and Sargant in the 1970s.

Finally, the static partial equilibrium model from Section 3, was augmented to a general equilibrium model, exemplified by a two-market model, to keep it as simple as possible. The interpretations from Section 3 completely generalized, and it was demonstrated how the general-partial equilibrium distinction has a natural correspondence to the CVAR. It was shown how the adequacy of the partial equilibrium analysis can be assessed in the CVAR, or whether general equilibrium is needed. This question is of course also completely central in economics, and empirical models that can model and clarify the presence and extent of general equilibrium feedbacks are indeed valuable.

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